Review Problems for Math1100 Exam 1

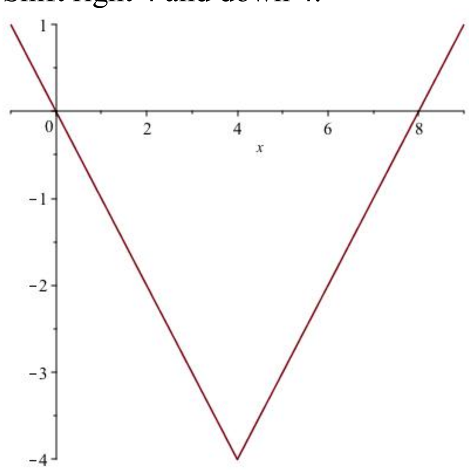
1. Find the standard form of the equation of the circle for which the center is (3,-2) and has a solution point (-1,1).
   $(x-3)^2+(y+2)^2=r^2$
   Plug in point to find r:
   $(-1-3)^2+(1+2)^2=(-4)^2+(3)^2=25=r^2$
   Standard Form:
   $(x-3)^2+(y+2)^2=25$

2. Find the x and y-intercepts, Domain and Range and carefully sketch the graph of the following equations.
   a. $y=|x-4|-4$

      x-int
      $0=|x-4|-4$
      $4=|x-4|$
      $x-4=4 \quad \& \quad x-4=-4$
      $x=8 \quad \& \quad x=0$
      $(8,0),(0,0)$

      y-int
      $y=|0-4|-4$
      $y=4-4=0$
      $(0,0)$

      Shift right 4 and down 4.



      $D:(-\infty,\infty)$
      $R:y\ge -4$

b. $y = (-x+1)^2 - 4$

x-int

$0 = (-x+1)^2 - 4$

$4 = (-x+1)^2$

$\pm 2 = -x+1$

$-1 \pm 2 = -x$

$1 \pm 2 = x$

$3, -1 = x$

$(3,0), (-1,0)$

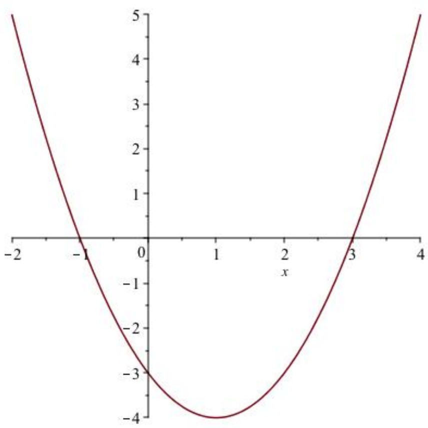
y-int

$y = (-0+1)^2 - 4$

$y = 1 - 4 = -3$

$(0,-3)$

Shift left 1, reflect over y-axis and down 4



$D: (-\infty, \infty)$

$R: y \geq -4$

3. Find the equation of the line that passes through the point (1,-2) and is parallel to $y = 1 - 2x$.

Parallel so slope is the same as the line $y = 1 - 2x$, m=-2. Use the point to solve for b.

$y = -2x + b$

$-2 = -2(1) + b$

$0 = b$

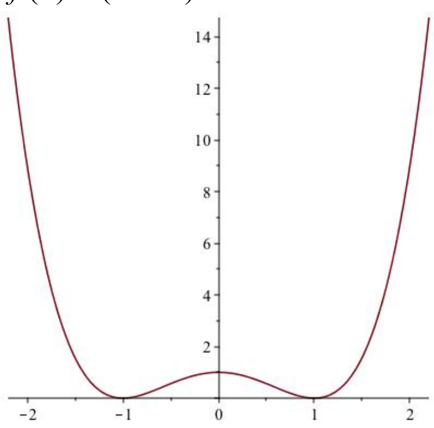
$y = -2x$

4. Determine the domain of the following functions.
   a. $f(x) = \sqrt{25 - x}$
   $25 - x \geq 0$
   $-x \geq -25$
   $D: x \leq 25$
   b. $f(x) = \dfrac{\sqrt{x-1}}{1-x}$
   The numerator has the restriction
   $x - 1 \geq 0$
   $x \geq 1$
   And the denominator has the restriction $x \neq 1$
   Put these together and $D: x > 1$

5. Use Is the function even, odd or neither $f(x) = \dfrac{x}{x^2 + 1}$

$$f(-x) = \frac{-x}{(-x)^2 + 1} = \frac{-x}{x^2 + 1} = -f(x) \text{ ODD}$$

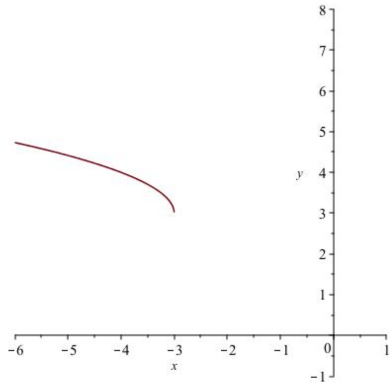
6. $f(x) = (x^2 - 1)^2$



   a. Determine the intervals where $f(x)$ is increasing, decreasing and constant.
   Increasing: $(-1,0)$ & $(1,\infty)$, Decreasing: $(-\infty,-1)$ & $(0,1)$
   b. Find the zeros of $f(x)$.
   X=-1&1
   c. Find any relative maximum or relative minimums.
   Relative mins: (-1,0) & (1,0), Relative max: (0,1)
   d. State the domain and range of $f(x)$.
   D: all real numbers, R: $y \geq 0$
   e. Determine whether the function is even, odd or neither.
   $f(-x) = ((-x)^2 - 1)^2 = (x^2 - 1)^2$ so even

7. Identify the transformation of the graph $f(x)$ and sketch $h(x)$.

   $f(x) = \sqrt{x} \qquad h(x) = \sqrt{-x-3} + 3$, Right 3, reflect y-axis, up 3

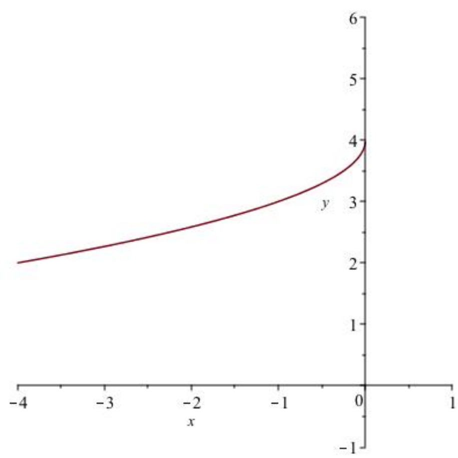


8. Write the function that result from taking the following actions in order. Sketch the resulting graph. Let $f(x) = \sqrt{x}$. Move $f(x)$ down 4 units; reflect about the x-axis and reflect about the y-axis.

   Move down 4: $f(x) = \sqrt{x} - 4$

   Reflect x-axis: $f(x) = -(\sqrt{x} - 4) = -\sqrt{x} + 4$

   Reflect y-axis: $f(x) = -\sqrt{-x} + 4$



9. Evaluate the function as indicated.

   $f(x) = 5x + 1$

   a. $f(-4) = 5(-4) + 1 = -20 + 1 = -19$

   b. $f(t^2) = 5(t^2) + 1 = 5t^2 + 1$

   c. $f(x+1) = 5(x+1) + 1 = 5x + 5 + 1 = 5x + 6$

   d. $\dfrac{f(x+h) - f(x)}{h}$

   $$= \frac{5(x+h) + 1 - (5x+1)}{h} = \frac{5(x+h) + 1 - 5x - 1}{h} = \frac{5x + 5h + 1 - 5x - 1}{h} = \frac{5h}{h} = 5$$

10. Solve the following systems of equations.

$$\begin{Bmatrix} x - y = 0 \\ 2x + y = 0 \end{Bmatrix}$$

Multiply row 1 by -2 and add the two rows and solve for y:

$$\begin{Bmatrix} -2x + 2y = 0 \\ 2x + y = 0 \end{Bmatrix} \Rightarrow 3y = 0 \Rightarrow y = 0$$

Use value for y to solve

$$x - y = 0$$
$$x - 0 = 0$$
$$x = 0$$

11. Find $(f+g)(x), (f-g)(x), (fg)(x), (f/g)(x), f \circ g, g \circ f$ for each of the following functions. Find the domain for $f \circ g, g \circ f$.

$$f(x) = x + 2 \quad g(x) = \sqrt{x+1}$$
$$(f+g)(x) = x + 2 + \sqrt{x+1}$$
$$(f-g)(x) = x + 2 - \sqrt{x+1}$$
$$(fg)(x) = (x+2)\sqrt{x+1} = x\sqrt{x+1} + 2\sqrt{x+1}$$
$$(f/g)(x) = \frac{x+2}{\sqrt{x+1}}, x > -1$$
$$f \circ g = \sqrt{x+1} + 2, D: x \geq -1$$
$$g \circ f = \sqrt{x+2+1} = \sqrt{x+3}, D: x \geq -3$$

12. Find $f^{-1}(x)$ and verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

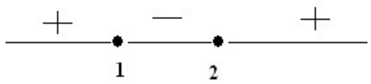
$$f(x) = \sqrt{x+1}$$
$$y = \sqrt{x+1}$$
$$x = \sqrt{y+1}$$
$$x^2 = y + 1$$
$$x^2 - 1 = y$$
$$f^{-1}(x) = x^2 - 1$$

Check

$$f(f^{-1}(x)) = \sqrt{x^2 - 1 + 1} = \sqrt{x^2} = x$$
$$f^{-1}(f(x)) = (\sqrt{x+1})^2 - 1 = x + 1 - 1 = x$$

13. Solve $x^2 - 3x + 2 \geq 0$

$(x-2)(x-1) \geq 0$



$(-\infty,1)$ $f(0) = 3$

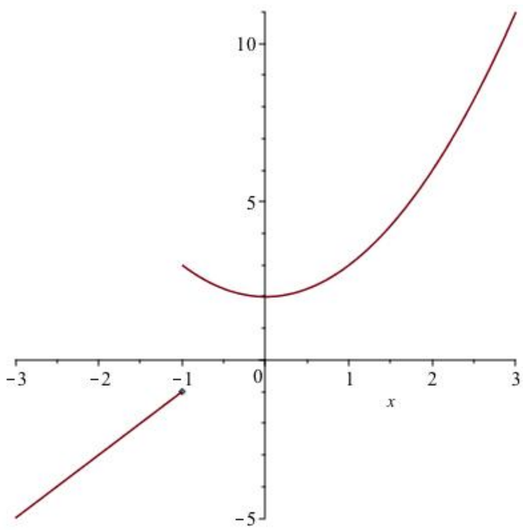
$(1,2)$ $f(3/2) = 9/4 - 9/2 + 2 = 9/4 - 18/4 + 8/4 = -1/4$

$(2,\infty)$ $f(3) = 9 - 9 + 2 = 2$

Solution: $(-\infty,1] \cup [2,\infty)$

14. Sketch the following systems of equations.

$$f(x)\begin{Bmatrix} 2x+1 & x \leq -1 \\ x^2+2 & x > -1 \end{Bmatrix}$$



15. Let $f(x) = \sqrt{x+1}$

a) Determine the average rate of change from $x_1 = -1$ to $x_2 = 8$.

$f(-1) = 0$ and $f(8) = 3$

$$ARC = \frac{f(8) - f(-1)}{8 - -1} = \frac{3-0}{9} = \frac{1}{3}$$

b) Write the equation of the secant line between $f(-1)$ and $f(8)$.

$$y = \frac{1}{3}x + b$$

$$0 = \frac{1}{3}(-1) + b \Rightarrow \frac{1}{3} = b$$

$$y = \frac{1}{3}x + \frac{1}{3}$$