MATH 3323 Linear Algebra
Problem Set 4
Due: March 23, 2020

On separate sheets of paper please solve all the problems below.

1. Consider the set of all ordered pairs of real numbers (i.e. R^2) with the following operations of "addition" and "scalar multiplication":
   $(x_1, y_1) + (x_2, y_2) = (y_1 + y_2, x_1 + x_2)$ and $k(x, y) = (kx, ky)$.
   Decide whether or not each of the vector space axioms is true for R^2 with these operations. **Go through all 10 axioms**. If an axiom is true, prove it. If an axiom is false, give a concrete counterexample (with numbers/vectors) showing it is false.

2. Determine which if the following are subspaces of P_2 (the vector space of all polynomials of degree less than or equal to two).
   a) The set of all polynomials with $a_0 = 0$ (Note: form $p(x) = a_1 x + a_2 x^2$).
   b) The set of all polynomials $p(x) = a_0 + a_1 x + a_2 x^2$ satisfying $p(1) = 1$.

3. Let V denote the set of all 2x2 matrices, and let W be the subset of V consisting of all 2x2 matrices having trace zero. Is W a subspace of V? Prove your answer. Feel free to use known facts about trace.

4. Show that vectors $v_1 = (2,2,2)$, $v_2 = (0,0,3)$ and $v_3 = (0,1,1)$ span R^3, and express the vector $(-1,2,0)$ as the linear combination of the three vectors.

5. Determine if the following are linearly independent subsets:
   a) Determine whether or not vectors (1,-1,1,1), (3,0,1,1), (7,-1,2,1) form a linearly independent subset of R^4.
   b) Let $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$. Do A, B, and C form a linearly independent subset of M_{2x2}?
   c) Determine if $5, x^2 - 6x, (3-x)^2$ form a linearly independent subset of $F(-\infty, \infty)$.

6. Are the following bases? Why or why not.
   a) {(1,0,2), (1,2,3), (-2,1,1)} in R^3?
   b) $\{x^3 + 2x^2, x + 3, -2\}$ in P_3?
   c) {(1,-1), (-2,2)} in R^2?

7. Determine the dimension of and basis for the solution space of the system:

$$x_1 - 2x_2 - x_3 = 0$$
$$2x_1 + x_2 + 3x_3 = 0$$

8. For each of the following, find the dimension of the subspace of the given vector space:

a) Vector space: R^3; subspace: all vectors of the form $\begin{bmatrix} a-b \\ b \\ a+b \end{bmatrix}$

b) Vector space: R^3; subspace: Span{(2,3,0),(1,0,3),(3,3,3),(-1,-3,3)}