MATH 3323 Linear Algebra
Problem Set 5
Due April 3, 2020

1. Let $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

   a) Find the general solution to $Ax = 0$
   b) Find a basis for the row space of A.
   c) Find a basis for the column space of A.
   d) Find a basis for the nullspace of A.
   e) Find rank(A)
   f) Find nullity(A)

2. Consider the bases $B = [u_1, u_2]$ and $B' = [v_1, v_2]$ for R^2 where

   $u_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, u_2 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}, v_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

   a) Find the transition matrix from B' to B.
   b) Find the transition matrix from B to B'.
   c) Compute the coordinate vector $[w]_B$ where $[w]_{B'} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$

3. Consider the differential equation $y''-4y'+5y = 0$

   a) Verify that each, $\{e^{2x}\sin x, e^{2x}\cos x\}$, is a solution to the differential equation.
   b) Test if the solutions are linearly independent (use Wronskian).
   c) If the set is linearly independent, write the general solution of the differential equation.

4. Let $u = (2,-1,3)$, $v = (1,-4,1)$, $w = (1,1,2)$. Find each of the following:

   a) $2u - v$
   b) The angle between u and v.
   c) $\|2u - 3v\|$
   d) A unit vector having the same direction as w.
   e) $proj_u v$
   f) $w \cdot (u - 2w)$
   g) $d(u, v)$

5. Compute the inner product

   a. $\langle u, v \rangle$: $u = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix}, v = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$

   b. $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$ with $f(x) = x$ and $g(x) = e^x$.

6. Find the k values so that u and v are orthogonal: $u = (k, k, 1)$, $v = (k, 5, 6)$

7. Let $v_1 = (1, -1, 2, -1)$, $v_2 = (-2, 2, 3, 2)$, $v_3 = (1, 2, 0, -1)$, $v_4 = (1, 0, 0, 1)$
    a. Verify that $\{v_1, v_2, v_3, v_4\}$ form an orthogonal basis for R^4.
    b. Turn $\{v_1, v_2, v_3, v_4\}$ into an orthonormal basis.
    c. Express (1,1,1,1) as a linear combination of $\{v_1, v_2, v_3, v_4\}$.

8. Let $u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$, $u_3 = (1, 2, 1)$, use Gram-Schmidt process to find orthogonal basis for the subspace spanned by u_1, u_2, u_3