

Solutions

1. Let $f(x) = x^2 + 6x + 5$

a) Determine the x & y-intercepts.

$$0 = (x+1)(x+5)$$

$$x = -1, \quad x = -5$$

$$(-1,0) \quad (-5,0)$$

b) Write the quadratic equation in standard form.

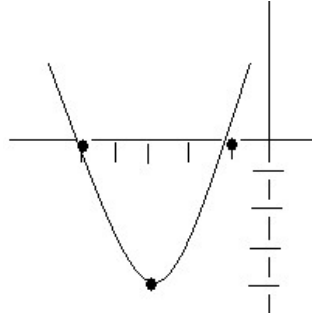
$$f(x) = x^2 + 6x + 9 - 9 + 5$$

$$f(x) = (x+3)(x+3) - 4$$

$$f(x) = (x+3)^2 - 4$$

c) Find the vertex of the quadratic function. $(-3,-4)$

d) Sketch the graph.



2. Let $f(x) = \frac{x^2 + 1}{x - 1}$. Find all asymptotes (vertical, horizontal and slant).

v.a.: $x - 1 = 0$
 $x = 1$

h.a.: none

slant: $x - 1 \overline{) \frac{x^2 + 0x + 1}{x^2 + 0x + 1}}$ $y = x + 1$

$$-(x^2 - x)$$

$$x + 1$$

$$-(x - 1)$$

$$2$$

3. Let $f(x) = \ln(x+1) - 1$

a) Find the domain of $f(x)$.

$$x+1 > 0$$

$$D: x > -1$$

b) Find the x & y-intercept of $f(x)$.

x-int

y-int

$$0 = \ln(x+1) - 1$$

$$y = \ln(0+1) - 1$$

$$1 = \ln(x+1)$$

$$y = \ln(1) - 1$$

$$e^1 = e^{\ln(x+1)}$$

$$y = 0 - 1$$

$$e = x+1$$

$$y = -1$$

$$e-1 = x$$

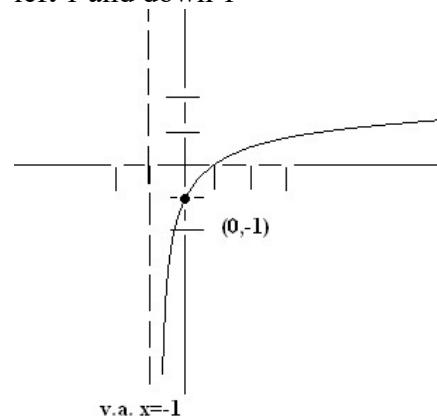
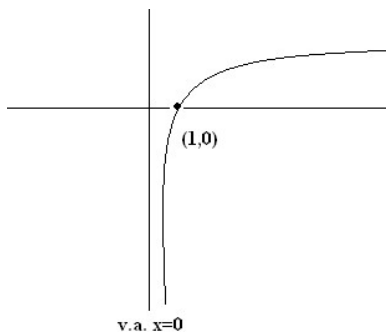
$$(0, -1)$$

$$(e-1, 0)$$

c) Sketch $f(x)$ (label the reference point and any asymptotes).

original

left 1 and down 1



4. Let $f(x) = -e^{x-2} + 4$

a) Find the domain of $f(x)$. All real numbers

b) Find the x & y-intercept of $f(x)$.

x-int

y-int

$$0 = -e^{x-2} + 4$$

$$-4 = -e^{x-2}$$

$$4 = e^{x-2}$$

$$y = -e^{0-2} + 4$$

$$\ln(4) = \ln(e^{x-2})$$

$$y = -e^{-2} + 4$$

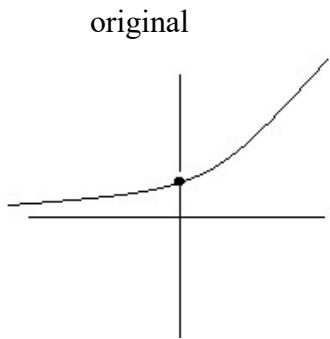
$$\ln(4) = x - 2$$

$$(0, -e^{-2} + 4)$$

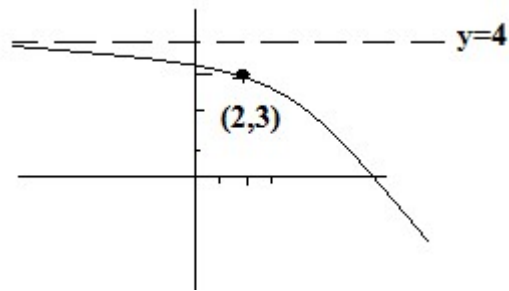
$$\ln(4) + 2 = x$$

$$(\ln(4) + 2, 0)$$

- c) Sketch $f(x)$ (label the reference point and any asymptotes).



right 2, reflect x and up 3



5. Let $f(x) = 2x^3 + 3x^2 - 3x - 2$

- a) List the possible rational zeros of $f(x)$.

$$\frac{\pm 1, \pm 2}{\pm 1, \pm 2} = \pm 1, \pm \frac{1}{2}, \pm 2$$

- b) Show that $x - 1$ is a factor of $f(x)$.

$$\begin{array}{r} 1 \overline{) 2 \ 3 \ -3 \ -2} \\ \underline{2 \ 5 \ 2} \end{array}$$

$$2 \ 5 \ 2 \ | \ 0 \text{ remainder} = 0$$

- c) Factor $f(x)$ completely.

$$f(x) = (x - 1)(2x^2 + 5x + 2) = (x - 1)(2x + 1)(x + 2)$$

6. Let $f(x) = -(x + 1)^2(x - 1)^2$

- a) Apply the leading coefficient test.

$n = 4$, $a_n = -1$, so falls left and right

- b) Find the zeros of $f(x)$ and their multiplicity.

$x = -1$, multiplicity 2, $x = 1$, multiplicity 2

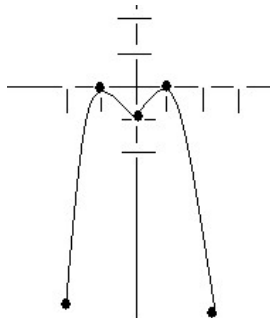
- c) Plot sufficient solution points.

$$(-\infty, -1) \quad f(-2) = -9$$

$$(-1, 1) \quad f(0) = -1$$

$$(1, \infty) \quad f(2) = -9$$

- d) Sketch $f(x)$.



7. Let $f(x) = \frac{2x+2}{x-1}$

a) Identify the x & y-intercepts.

x-int

y-int

$$0 = 2x + 2$$

$$-2 = 2x$$

$$-1 = x$$

$$(-1, 0)$$

$$y = \frac{2(0)+2}{0-1} = \frac{2}{-1} = -2$$

$$(0, -2)$$

b) Find all asymptotes (vertical, horizontal and slant).

v.a.: $x - 1 = 0$

h.a. degree of numerator = denominator, so

$$x = 1$$

$$y = \frac{2}{1} = 2$$

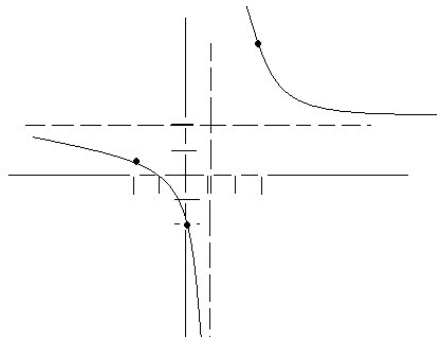
c) Plot sufficient solution points.

$$(-\infty, -1) \quad f(-2) = 2/3$$

$$(-1, 1) \quad f(0) = -2$$

$$(1, \infty) \quad f(2) = 6$$

d) Sketch the graph of $f(x)$



8. Solve for x.

a) $4^{x-3} = 16$

$$4^{x-3} = 4^2$$

$$x - 3 = 2$$

$$x = 5$$

$$\text{b) } \ln(3x-2) = 0$$

$$e^{\ln(3x-2)} = e^0$$

$$3x-2=1$$

$$3x=3$$

$$x=1$$

$$\text{c) } e^{2x} - 5e^x + 4 = 0$$

$$(e^x - 4)(e^x - 1) = 0$$

$$e^x - 4 = 0$$

$$e^x - 1 = 0$$

$$e^x = 4$$

$$e^x = 1$$

$$\ln(e^x) = \ln 4$$

$$\ln(e^x) = \ln 1$$

$$x = \ln 4$$

$$x = 0$$

$$\text{d) } 2\log_4 x = \log_4(2x) + \log_4(x-1)$$

$$\log_4 x^2 = \log_4(2x)(x-1)$$

$$x^2 = (2x)(x-1)$$

$$x^2 = 2x^2 - 2x$$

$$0 = x^2 - 2x = x(x-2)$$

$$x = 0, x = 2$$

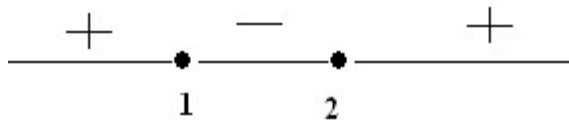
$\ln$ can't be zero, so only $x = 2$ works.

$$9. \text{ Solve } x^2 - 3x + 2 \leq 0$$

$$(x-2)(x-1) \leq 0$$

$$(x-2)(x-1) = 0$$

$$x = 2, x = 1$$



$$(-\infty, 1) \quad f(0) = 3$$

$$(1, 2) \quad f(3/2) = 9/4 - 9/2 + 2 = 9/4 - 18/4 + 8/4 = -1/4$$

$$(2, \infty) \quad f(3) = 9 - 9 + 2 = 2$$

Solution: $[1, 2]$