Review Exam 1

1. Evaluate the following limits if they exist. If not, explain.

a) $\lim\limits_{x \to 4}(-x^3 + 4x - 7)$

b) $\lim\limits_{x \to \pi}(3x\cos(x))$

c) $\lim\limits_{x \to 4}\left(\dfrac{4-x}{3x-12}\right)$

d) $\lim\limits_{x \to 4}(-7)$

e) $\lim\limits_{x \to -3}\dfrac{\dfrac{1}{x}+\dfrac{1}{3}}{x+3}$

f) $\lim\limits_{x \to 1}\dfrac{x-1}{\sqrt{x+8}-3}$

g) $\lim\limits_{t \to 4}\left(\dfrac{t^2-2t-8}{t^2-5t+4}\right)$

h) $\lim\limits_{z \to 4}\left(\sqrt{z^2-16}\right)$

i) $\lim\limits_{\vartheta \to 0}\left(\dfrac{\sin(4\vartheta)}{\vartheta}\right)$

j) $\lim\limits_{\vartheta \to 0}\left(\dfrac{\sin(4\vartheta)}{\sin(3\vartheta)}\right)$

k) $\lim\limits_{\vartheta \to 0}\left(\dfrac{\sin(\vartheta)}{\tan(\vartheta)}\right)$

l) $\lim\limits_{x \to 5}\left(7+3x+\dfrac{6}{(x-5)^2}\right)$

m) $\lim\limits_{x \to 7^+}\left(\sqrt{x-7}\right)$

n) $\lim\limits_{x \to 0^-}\left(\dfrac{x+\sin(x)}{x}\right)$

o) $\lim\limits_{x \to -1}\left(\dfrac{x}{x+1}\right)$

p) $\lim\limits_{x \to -3}(x+3)^2 \sin\left(\dfrac{x-2}{x+3}\right)$

2. Evaluate the following limits. State whether or not the function has a horizontal asymptote at the given ∞.

a) $\displaystyle\lim_{x\to\infty}\frac{3}{x^9}$

b) $\displaystyle\lim_{x\to\infty}\frac{5x^7-2x+4}{1-x^6}$

c) $\displaystyle\lim_{x\to\infty}\frac{3x}{\sqrt{x^5}}$

d) $\displaystyle\lim_{x\to\infty}\frac{7x^4-9x^2+x^8}{3x^8-2x^5}$

e) $\displaystyle\lim_{x\to-\infty}\frac{1-5x}{\sqrt{x^2+2x}}$

f) $\displaystyle\lim_{x\to-\infty}\frac{x^2}{\sqrt{x+9x^4}}$

3. Let $f(x)=\begin{cases} x^3-2x+15 & x\le-3 \\ \dfrac{a}{x}-x^2+6 & -3<x\le-1 \\ bx-11 & x>-1 \end{cases}$

   a) Find the constants $a\,\&\,b$ such that $f(x)$ is continuous everywhere.
   b) Given the constants you found in part (a), is $f(x)$ differentiable everywhere?

4. Find the discontinuities of the following functions. Which of the discontinuities are removable? For each function, are there any vertical asymptotes? Explain.

a) $f(x)=\dfrac{\sin x}{x}$

b) $f(x)=\dfrac{1}{x-6}$

c) $f(x)=\dfrac{x-3}{x^2-2x-3}$

d) $f(x)=\dfrac{x+8}{x^2+2}$

e) $f(x)=\tan x$

f) $f(x)=\dfrac{|x+1|}{x+1}$

5. Use the definition of a derivative to find y' for each of the following functions
    a) $y = 7 - 3x$
    b) $y = 9x - x^2 + 4$
    c) $y = \dfrac{3}{x - 2}$
    d) $y = \sqrt{7 + x}$
    e) $y = x + \dfrac{1}{x^2}$

6. If $g(x) = 2x^3 - \dfrac{12}{x}$, show that $g(x)$ has a zero on $[1,2]$.

7. Find the equation of the tangent line for $g(x) = \dfrac{6x^7 - 3x + 4}{x^2}$ at $x = 1$.

8. Find the values of x for which $h(x) = x^3 - 4x^2 - 11x + 5$ has a horizontal tangent.

9. Find $f'(x)$ for the following:
    a) $f(x) = 3x^4 - 9x^{5/3} + \dfrac{21}{x^2}$
    b) $f(x) = 4\sin x - 6x^2$
    c) $f(x) = x^\pi$
    d) $f(x) = \sqrt[5]{x^7} + \dfrac{2}{(2x)^6} - 99$
    e) $f(x) = 5x^2 \left(9 - x^5 + \dfrac{1}{x^7} \right)$
    f) $f(x) = \sqrt{x}\tan(x)$
    g) $f(x) = \dfrac{x^2 - 8}{4x + 2}$
    h) $f(x) = 3^x \ln(x + 3)$
    i) $f(x) = \dfrac{\ln(5x + 1)}{x^2}$
    j) $f(x) = \dfrac{5 - \dfrac{8}{x}}{4x}$
    k) $f(x) = x^2 \left(3 - \dfrac{5x - 8}{x + 2} \right)$
    l) $f(x) = (8e^x - 7x^4 + 2)^{78}$
    m) $f(x) = x^2 (6x^7 - 4x)^{3/7}$
    n) $f(x) = \log_5(x^3 - 9x + 1)$

o) $f(x) = e^{1-x^3} - e^{1/x}$

p) $f(x) = \sin(x^3 - 7x)$

q) $f(x) = \sin(\cos(4x^3 - 7x + 4))$

r) $f(x) = \left(\dfrac{x^2 - 8}{\cos(4x)} \right)^{1/2}$

s) $f(x) = \ln \sqrt[3]{\dfrac{x^2 - 1}{x^2 + 1}}$

t) $f(x) = \ln(\ln x)$

u) $f(x) = \sqrt{(3x^2 + 2)\sqrt{6x - 7}}$

v) $f(x) = \dfrac{(x^2 + 3)^{2/3} (3x - 4)^4}{\sqrt{x}}$

w) $f(x) = \arctan(3x^2 + 5x)$

x) $f(x) = x^{x^2}$

10. Find $\dfrac{dy}{dx}$ for the following:

a) $y = (8 + \cos(x^4) + \sqrt{x})^{12}$

b) $y^3 + x^2 = 9$

c) $\cos(x + y^2) = 3x^7 - 9x + 1$

d) $yx^2 - \sin y = 0$

e) $\sec(y) = x$

f) $\sqrt[5]{y^2 - x} + x = 9y$

11. Find y'' for $f(x) = x^2 \cos x + \sin x - 7x$

12. If $f(x) = 6x^3 - 9x^{23} + 67$, how many derivatives do you need to take to get a derivative which is 0?

13. If the position function of a free-falling object is given by
$s(t) = -16t^2 + 12t + 40$ ft, find the requested information.

a) Find $v(t)$

b) Find $a(t)$

c) What is the initial position? What is the initial velocity?

d) How long does it take to hit the ground?

e) Find the highest point the ball will achieve.

f) Find the velocity when the ball hits the ground.

g) Find the average velocity on the time interval $[1/2, 1]$

h) Find the instantaneous velocity at $t = 3/4$ seconds.

14. Given $f(x) = 4x^5 - \dfrac{1}{x^3}$. Find the slope of the tangent line at the point $(3,1)$ on the graph of $f^{-1}(x)$.

15. Gas is being pumped into a spherical balloon at a rate $5\,ft^3/\min$. Find the rate at which the radius is changing when the diameter is 18in.

16. A girl starts at a point A and runs east at the rate of $10\,ft/\sec$. One minute later, another girl starts at A and runs north at a rate of $8\,ft/\sec$. At what rate is the distance between them changing 1 minute after the second girl starts?

17. A man walks along a straight path at a speed of $4\,ft/\sec$. A searchlight is located on the ground 20 feet from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 feet from the point on the path closest to the searchlight?