MATH 3323 Linear Algebra
Practice Problems Exam 1

1. Solve the system

$$\begin{cases} \quad x_2 + 4x_3 = -5 \\ x_1 + 3x_2 + 5x_3 = -2 \\ 3x_1 + 7x_2 + 7x_3 = 4 \end{cases}$$

2. Give an example of an inconsistent system with two equations and three unknowns.

3. Let $A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Show that $Ax = b$ does not have a solution for all possible b and determine the set of all b for which $Ax = b$ does have a solution.

4. Determine if the following system has a non-trivial solution.

$$\begin{cases} x_1 - 3x_2 + 7x_3 = 0 \\ -2x_1 + x_2 - 4x_3 = 0 \\ x_1 + 2x_2 + 9x_3 = 0 \end{cases}$$

5. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$. Compute $(3A + B)^T$, CA, $(C^{-1})^T$, $(B - A)C$ and $tr(C^3)$ if possible. If not, explain.

6. If the second column of B is all zeros, what can you say about the second column of AB? Explain your reasoning.

7. Is the following equality true? $(A + B)(A - B) = A^2 - B^2$

8. Show that if $ABCD$ is invertible, then $(ABCD)^{-1} = D^{-1}C^{-1}B^{-1}A^{-1}$.

9. Find the inverse of $A = \begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$ if it exists. If not, explain.

10. Compute the following determinant using cofactor expansion. $\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix}$

11. Compute the following determinants using known theorems (not cofactor expansion). Justify your answers

$$\begin{vmatrix} 3 & 5 & -8 & 4 \\ 0 & -2 & 3 & -7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 2 & 4 & -2 \\ 1 & 3 & -1 & 4 \\ 0 & 0 & 3 & 6 \\ 1 & 3 & -1 & 6 \end{vmatrix}$$

12. Let $A = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$

a) Create an elementary matrix E_1 that will swap R_2 and R_4.
b) Create an elementary matrix E_2 that multiplies R_3 by -6.
c) Create an elementary matrix E_3 that adds 4 R_2 to R_1.
d) What is the result of multiplying E_1, E_2 and E_3 (in that order) to A?

13. Solve the system using LU-factorization. State your L and your U matrices.

$$2x_1 + x_2 = 4$$
$$x_2 - x_3 = 8$$
$$-2x_1 + x_2 + x_3 = -8$$

14. Let $A = \begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix}$

a) Find the matrix of cofactors
b) Find the adjoint of A
c) Using your answers from about (including what you found in problem 10), write down A^{-1}

15. Suppose A and B are 3x3 matrices, and that $\det(A) = 2$ and $\det(B) = -3$. Find each of the following:
    a) $\det(BA)$
    b) $\det(3B)$
    c) $\det(A^{-1})$

16. If A is symmetric, prove $A - AA^T$ is symmetric.

17. Find the area of the triangle whose vertices are (1,2), (3,-4) and (-5,-1)

18. Find the equation of the plane passing through the points (2,3,1), (-1,2,0) and (0,2,2)