MATH 3323 Linear Algebra
Problem Set 1
Due February 3, 2020

On separate sheets of paper please solve all the problems below. Show your work.

1. Solve the following systems:

a) $\begin{cases} x_1 + 2x_2 + 3x_3 = 6 \\ -x_1 - 2x_2 - 2x_3 = -5 \\ 3x_1 + 5x_2 + 4x_3 = 11 \end{cases}$ b) $\begin{cases} x_1 + 3x_2 + x_3 + x_4 = 3 \\ 2x_1 - 2x_2 + x_3 + 2x_4 = 8 \\ x_1 - 5x_2 + x_4 = 5 \end{cases}$

2. Consider the system of equations

$$x + y + 2z = a$$
$$x + z = b$$
$$2x + y + 3z = c$$

Show that for this system to be consistent, the constants a, b, and c must satisfy $c = a + b$.

3. Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 & 4 \\ -2 & 1 & 4 \end{bmatrix}$

Calculate:

a) CA
b) AB
c) B^T
d) $B^T CA + I$

4. A large field contains a certain number of chickens and pigs. A farmer counts that there are 130 head and 464 feet. How many chickens and how many pigs are there? Set up linear equations (there are two) to solve.

5. Prove: If A and B are nxn matrices, then $tr(A+B) = tr(A) + tr(B)$.

6. Indicate whether the statement is always true or sometimes false. If true, prove it is. If false, show one counterexample.

a) The expression $tr(AA^T)$ and $tr(A^T A)$ are always defined, regardless of the size of A.
b) $tr(AA^T) = tr(A^T A)$ for every matrix A.
c) If the first column of A has all zeros, then so does the first column of every product AB.
d) If the first rows of A has all zeros, then so does the first row of every product AB.