MATH 3323 Linear Algebra
Problem Set 2
Due February 12, 2020

On separate sheets of paper please solve all the problems below.

1. Consider the system $\begin{cases} -3x & +4y & =2 \\ 4x & +5y & =1 \end{cases}$.
   a) Give the matrix equation that is equivalent to this system.
   b) Solve the system by inverting the coefficient matrix.

2. Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$. Verify by checking $AA^{-1} = A^{-1}A = I$.

3. Solve the system using LU-factorization. State your L and your U matrices.
   $2x_1 + x_2 = 4_1$
   $x_2 - x_3 = 8$
   $-2x_1 + x_2 + x_3 = -8$

4. Find $\begin{bmatrix} \mathrm{O} & B \\ C & D \end{bmatrix}^{-1}$. Here O denotes a zero matrix. Assume O, B, C, and D are all square matrices of the same size. Indicate which matrices must be assumed invertible. Show all of your work.

5. a) Show that a matrix with a row of zeros cannot have an inverse.
   b) Show that matrix with a column of all zeros cannot have an inverse.

6. Indicate whether the statement is always true or sometimes false. If true, prove it is. If false, show one counterexample.
   a) Every square matrix can be expressed as a product of elementary matrices.
   b) The product of two elementary matrices is an elementary matrix.
   c) If A is invertible and a multiple of the first row of A is added to the second row, then the resulting matrix is invertible.
   d) If A is invertible and AB = 0, then it must be true that B = 0.

7. Find the conditions that the b's must satisfy for the system to be consistent.
   $x_1 - 2x_2 - x_3 = b_1$
   $-4x_1 + 5x_2 + 2x_3 = b_2$
   $-4x_1 + 7x_2 + 4x_3 = b_3$