MATH 3323 Linear Algebra
Problem Set 3
Due February 24, 2020

1. Let $A = \begin{bmatrix} 1 & 8 & 7 \\ 2 & 10 & 6 \\ 3 & 4 & 5 \end{bmatrix}$

   a) Find the $\det(A)$
   b) Find the matrix of cofactors of A.
   c) Find the adjoint of A.
   d) Using your answers from parts a) and c), write down A^{-1}.

2. Suppose A, B, and C are 2x2 matrices, and that $\det(A) = -3$, $\det(B) = 2$, and $\det(C) = 1/2$. Find each of the following:

   a) $\det(AB)$
   b) $\det(C^T)$
   c) $\det(B^{-1})$
   d) $\det(2A)$
   e) $\det(BA^{-2}C)$

3. Find $\det \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 1 & 3 & 0 \\ 0 & 3 & 1 & 3 \\ 0 & 0 & 3 & 1 \end{bmatrix}$.

4. Use elementary row or column operations (not cofactor expansion!) to find each of the following determinants. Justify your answers:

   a) $\det \begin{bmatrix} 1 & -2 & 3 \\ 7 & 1 & 4 \\ -2 & 4 & -6 \end{bmatrix}$

   b) $\det \begin{bmatrix} 2 & 4 & 3 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix}$

   c) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$, find $\begin{vmatrix} -3d & -3e & -3f \\ a & b & c \\ g+5a & h+5b & i+5c \end{vmatrix}$

5. Use Cramer's Rule to solve the system:

$$\begin{cases} 3x + 2y = 7 \\ x - y = 4 \end{cases}$$

6. Determine the value(s) of k for which the matrix is invertible:

$$\begin{bmatrix} -1 & 0 & 1 \\ 1 & k & -1 \\ -1 & 1 & 1 \end{bmatrix}.$$

7. Suppose that A is an invertible nxn matrix. Prove that $\det(adj(A)) = (\det(A))^{n-1}$.